Turbulent Boundary-Layer Characteristics of Compliant Surfaces

Edward F. Blick* and Robert R. Walters†
University of Oklahoma, Norman, Okla.

A hot-wire anemometer study was made of the turbulent boundary layer on a compliant coating. A compliant coated flat plate was made by covering a $\frac{5}{16}$-in.-deep reservoir of fluid with a thin sheet of polyvinyl chloride. This compliant test plate which measured $26\frac{1}{8}$ in. × $8\frac{1}{8}$ in. was inserted flush in the floor of a low-speed wind tunnel. The hot-wire anemometer was used to measure velocity profiles, Reynolds stresses and turbulence intensity in the boundary layer. All tests were run at 38 fps and the skin tension, skin thickness, and reservoir fluid properties were varied during the tests. The universal velocity profile of the compliant coating indicated no change in mixing length from that of a hard plate but seemed to indicate a thicker laminar sublayer. The Reynolds stress and turbulence intensity were smaller for the compliant coating than for the hard plate and they seemed to correlate with previously measured skin-friction reductions.

Nomenclature

C_f, C_F = local and average skin-friction coefficient, respectively
h = membrane thickness
I = turbulence intensity $\equiv \overline{(u_1^2)}^{1/2}/U_1$
$\overline{q^2}$ = $\overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}$
T_x, T_z = tension in longitudinal (streamwise) and lateral direction, respectively
U_1 = freestream velocity
u_1, u_2, u_3 = instantaneous fluctuating components of velocity in the streamwise, normal to wall and parallel to wall, respectively
X_0 = distance (in the flow direction) from the leading edge of the test plate
Y = distance above the plate
ρ = density
ν = kinematic viscosity

Introduction

In the last few years, considerable interest has been generated in the area of drag reduction by compliant coatings or flabby skins. Much of this interest was stimulated by the work of Kramer,[1-3] which indicated a reduction of drag on bodies towed in water by coating the surfaces of the bodies with a compliant coating. The coating was inspired by the flabby skin of the dolphin, which, according to Kramer,[3] is responsible for its high speed. Kramer's theory is that the dolphin's flabby skin damps the near-wall flow disturbances which retards transition and results in a larger area of laminar flow (with low values of skin friction) over the dolphin. Theoretical analyses by Benjamin,[4] Landahl,[5] and Kaplan[6] indicate that the stability of the laminar boundary layer can be altered by the use of compliant boundaries.

The effect of flabby skins in the transition and turbulent region has received a very limited amount of attention. Karplus[7] investigated the scale and degree of turbulence for water flowing over stretched mylar film backed with different damping fluids. He found that in his tests, turbulence sets in sooner for the flexible wall than for a solid wall, but grows to its final value more slowly. The damping fluid behind the film made the surface appear more like a solid surface as the viscosity was increased.

As promising as the initial results of Kramer were, unfortunately, only a few experimenters since then have been able to measure a reduction in skin drag using compliant coatings. Lauffer and Maestrello,[8] in an experimental study of turbulent airflow in channels lined with the thin steel, aluminum, mylar, and fabric membranes, detected no significant skin-friction reduction. Smith,[9] using water flow through pipes with annular coatings of elastic gel, could not detect any reduction in skin friction. Gregory and Love[10] investigated the effects of surface flexibility on the drag of a 5-ft chord airfoil utilizing a foamed sandwich flexible surface and they found no skin-friction reduction. Benjamin[11] has reported that Dinkelacker of the University of Southampton investigated the skin friction of flexible walls and found no decisive skin-friction reductions. Stephens[12] tested corrugated skins of 0.001-in. Melinex (fabricated by Gregory) and obtained no reduction. Ritter[13] found a drag reduction of 7–14% on a Kramer-type coating but the scatter of the data was so large that the results are questionable.

On the promising side were the results of Von Winkel and Barger,[14] Pelt,[15] Fisher and Blick,[16] and Looney and Blick[17]. Von Winkel and Barger[14] measured surface pressure fluctuations on a Kramer-type flexible skin and found a considerable reduction in the intensity of the fluctuations. Pelt[15] found that skin friction in pipe lines could be reduced by as much as 35% by lining the pipe with a flexible tube. The annular space between the flexible tube and pipe wall was filled with a variety of damping fluids (air, water, and solutions of glucose). The flexible tubes used were "Texin" tubes, "Tygon" tubes, and rubber tubes. Pelt found that Texin tubes gave the biggest skin-friction reduction. Fisher and Blick[16] measured the turbulence intensity over a compliant wall in the wake of a small fence in a wind tunnel. The compliant coating consisted of a 0.00225-in.-thick polyvinyl chloride membrane stretched over a 0.375-in.-deep reservoir of damping fluid. The damping fluids used were air, water, and SAE 30 weight automobile oil. It was observed that the compliant surface had lower turbulence intensities than the hard surface. The ability of the compliant coating to damp out the turbulence seemed to increase with an increase in viscosity of the damping fluid as oil was the best damping fluid followed by water, then air.

The experiments of Looney and Blick[17] indicated that it was possible to reduce the turbulent skin-friction drag by using a compliant surface of thin polyvinyl chloride material covering a fluid reservoir $\frac{5}{16}$ in. deep. Skin-friction reductions up to 50% were obtained at tunnel velocities of 38 fps and a Reynolds number of approximately 0.8×10^6.

Presented as Paper 67-128 at the AIAA 5th Aerospace Sciences Meeting, New York, January 23–26, 1967; submitted December 28, 1966; revision received September 29, 1967. This work was sponsored by the United States Army Research Office, Durham N. C. under Contract DA-31-124-ARO-D-349.

* Associate Professor, School of Aerospace and Mechanical Engineering. Member AIAA.

† Graduate Student, School of Aerospace and Mechanical Engineering. Student Member AIAA.

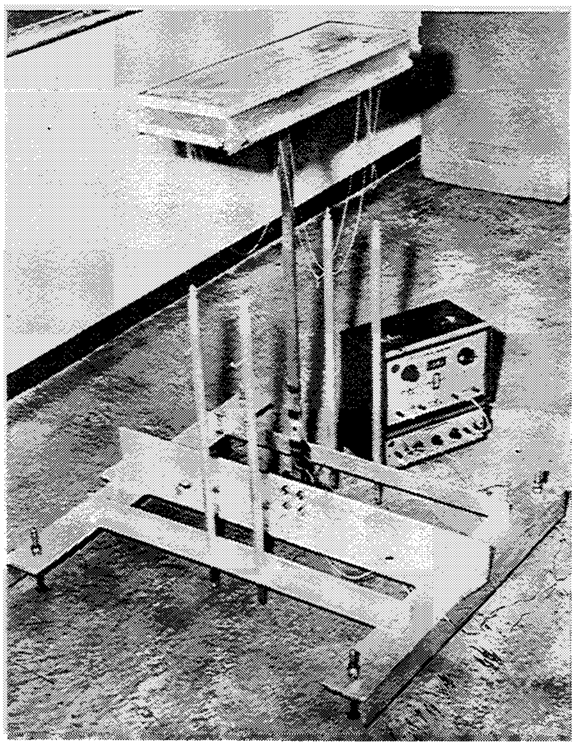

Fig. 1 Compliant-surface test panel.

The observations of Refs. 14–17 indicate that there may be a correlation between a reduction in turbulence intensities and a reduction in C_f. This intuitive approach to the phenomenon may be strengthened by noting that Von Karman's similarity hypothesis dictates a constant ratio between the Reynolds stresses and turbulence kinetic energy.[18] Klebanoff's data[19] showed $\rho\overline{u_1 u_2} \propto \rho\overline{q^2}$ to be approximately true over a major portion of the turbulent layer on a flat plate, where $\rho\overline{u_1 u_2}$ = Reynolds stress. Since $\overline{u_1} \propto \overline{u_2} \propto \overline{u_3}$, $\overline{q^2} \propto \overline{u_1}^2 \propto I^2$, and hence $\rho\overline{u_1 u_2} \propto I^2$. Since the shear stress in a turbulent boundary layer is the sum of laminar friction and the Reynolds stresses, it seems reasonable to expect that the first-order effect of a reduction in I would be the reduction of that portion of C_f which depends upon the Reynolds stress. The differences in the values of turbulent and laminar C_f are due primarily to the existence of the Reynolds stresses in the turbulent boundary layer; hence, one might conclude that

$$(C_{ff} - C_f')/(C_f'' - C_f') \propto (I_f/I_{hp})^2 \quad (1)$$

where C_{ff} is the local skin-friction coefficient of flexible wall, C_f'' and C_f' are the local turbulent and laminar skin-friction coefficients of the hard plate, respectively, and I_f and I_{hp} are the turbulence intensities of the flexible wall and the hard plate, respectively.

The test data of Looney and Blick[17] indicated that the skin friction was significantly lower for a flexible plastic skin (polyvinyl chloride) than for the hard plate in the wind-tunnel tests. If the hypothesis presented previously, concerning turbulence intensities and skin friction is correct, it seems reasonable to assume that the turbulence intensities would also be lower on the compliant, polyvinyl chloride skin. It was the intent of the authors to see if such a turbulence intensity reduction could in fact be detected in the wind tunnel and to investigate other turbulent boundary-layer characteristics of the compliant surface. The test facilities and test conditions reported on in this paper are exactly the same ones that were used by Looney and Blick,[17] as both projects were carried on simultaneously and shared the same compliant surface test plates and wind tunnel.

Experimental Apparatus

The wind tunnel used for this study and additional details of the experiment are described in detail in Ref. 20 or 21. The tunnel is a small open-circuit tunnel with a test section velocity of 38 fps, powered by a $1\frac{1}{2}$-hp, three-phase, constant-speed motor. The test section is $47\frac{1}{2}$ in. long by $19\frac{7}{8}$ in. wide by $13\frac{1}{8}$ in. high. One side of the test section was constructed of glass to permit observation during testing; the other side had two access doors.

The top of the tunnel was made of $\frac{1}{4}$-in. plywood and could be adjusted by means of screws so as to vary the cross-sectional area in the wind tunnel. The cross-sectional area was adjusted so that it increased slightly in the downstream direction and in effect cancelled out the decrease in effective flow area due to boundary-layer displacement growth. This resulted in a near-zero pressure gradient in the test section. In no case did the pressure gradients exceed 0.00021 psi/in. as measured by pressure taps in the floor of the tunnel and a micromanometer, model MM-3, manufactured by Flow Corp., Cambridge, Mass. This device is capable of monitoring four pressures simultaneously and measures air pressure changes to 0.000006 psi accuracy.

The compliant surface test plate was a floating panel flush with the floor of the wind tunnel (See Fig. 1). The floating panel was constructed of aluminum and measured $26\frac{1}{8}$ in. long by $8\frac{1}{8}$ in. wide and was constructed with $\frac{1}{2}$- by $\frac{5}{16}$-in. white pine wood retaining walls cemented around the top perimeter. This in effect made a reservoir to hold the damping fluid with interior dimensions of $25\frac{1}{8}$ in. long by $7\frac{1}{8}$ in. wide with a depth of $\frac{5}{16}$ in. The leading edge of the test plate was 28.2 in., measured along the floor from the wind-tunnel inlet.

The compliant or flexible surface was made by filling the reservoir with a fluid of known viscosity and then stretching a sheet of polyvinyl chloride across the reservoir. The polyvinyl chloride material (PVC), (commercial name, Clopay Frosty) was manufactured by the Hooker Chemical Corp. and thicknesses of 0.0035 and 0.0025 in. were used in the tests. This material had an extremely low Young's modulus of only 1400 psi and hence was very flexible. The skin was cut so that approximately 3 in. would hang over each side of the test plate. Two $\frac{1}{2}$- by $\frac{5}{16}$-in. white pine wood strips were clamped on the outer edge of each of the four sides of the skin. Fastened to each wood strip by a nylon cord was a weight retainer in which various known weights could be placed so that the tension on the skin could be varied in both the longitudinal and lateral directions. A thin film of Dow Corning silicone lubricant was placed on the top edge of the wood frame so that damping fluid leakage between the frame and the overlaid skin could be held to a minimum. When the membrane was in position over the reservoir and under tension, there was approximately a $\frac{1}{16}$ in. gap between the test plate and the edges of the wind-tunnel floor.

A stand was built to support a vertical cantilever beam which held the test plate in position. The main support

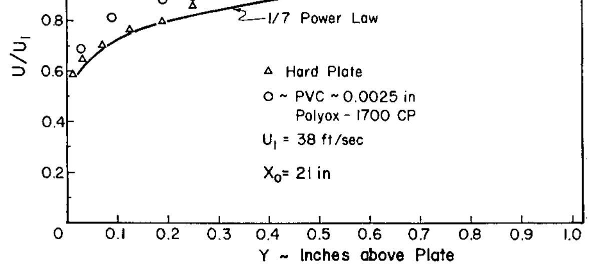

Fig. 2 Velocity profiles for flat plate at X_0 = 21 in.

consisted of three large 4-in. angle irons welded together to form an "I". The cantilever beam was fastened to the cross member and a $\frac{1}{2}$- × 6-in. bolt was placed in each corner of the other two angle irons, which formed the base legs, to provide a fine vertical adjustment for the flat plate when placed in the tunnel. The cantilever beam was instrumented with strain gages to measure the bending strain caused by the skin friction along the test plate. In order to keep the plate in a rigid condition when using the hot-wire anemometer above it, four supporting extension arms were placed on the support platform. These were made from two sizes of pipe, one sliding into the other, and welded to another cross-member on the platform. When skin-friction measurements were being taken, the stabilizer arms were lowered to free the plate so all forces would be transferred to the cantilever beam.

A hole slightly larger than the test plate was cut in the floor of the wind tunnel to accommodate the test plate. The rectangular PVC skin was laid over the test plate and weights were hung along all four sides until the desired longitudinal and lateral tension values were reached. Before each run the plate height and level were adjusted so that an over-all smooth tunnel floor existed.

A slot was cut in the top of the wind tunnel so that a hot-wire anemometer could be positioned anywhere along the center plane of the tunnel. The hot-wire anemometer was a Flow Corp. Model CCB two-channel system. The system components included a HWB-3 hot-wire anemometer, a HWI-3 sum and difference control unit, 12A-1 random signal voltmeter, 7KC low-pass filter, WHP-B probe with tungsten single wire, and a HSPX probe with TX tip using W-1 tungsten crosswires.

During the initial check-out of the hot-wire anemometer, it was discovered that velocity profiles in the boundary layer were definitely affected by airflow coming up through the slot between the test plate and the edge of the tunnel floor. This was corrected by placing a large plastic bag under and around the heavy steel support stand, and sealed to the underside of the wind tunnel. The bag was made of 5 sheets of 6-mil polyethylene each approximately 3 ft × 3 ft in size and sealed by means of zinc chromate vacuum putty. After the bag was installed, measurements of the velocity profile on the test plate appeared normal and matched very closely the $\frac{1}{7}$ velocity profile typical of turbulent boundary layers on flat plates (See Fig. 2).

The fluids that were used in the test plate reservoir included air, water, and solutions of water and polyethylene oxide, WSR-301 (manufactured by Union Carbide Co.). Polyethylene oxide commonly called "Polyox" is a water-soluble resin made of ethylene oxide polymers and has a molecular weight of approximately 4,000,000. It is possible to vary the viscosity of a water solution from around 50 CP (centipoise) with 0.2% resin by weight, to around 8000 CP with 1.5% resin by weight. Solution of Polyox in water are

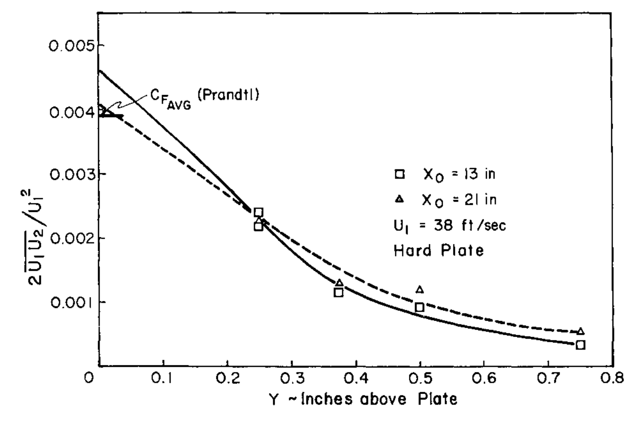

Fig. 3 Distribution of Reynolds stress on hard plate.

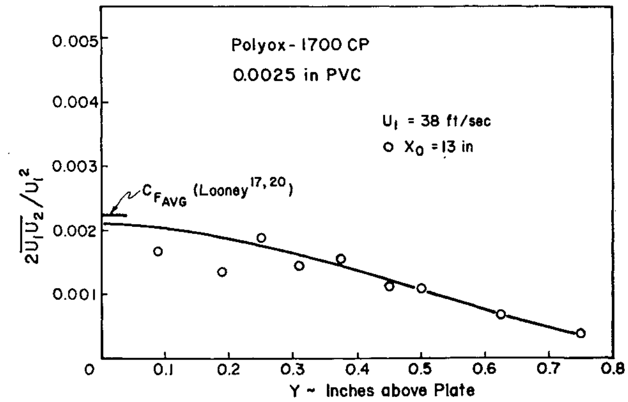

Fig. 4 Distribution of Reynolds stress on 0.0025-in. PVC skin with polyox 1700 CP as damping fluid.

speudoplastic at high-shear rates and this property becomes more pronounced as the concentration is increased. The viscosities of the Polyox water solution reported in this paper were measured at low-shear rates by a Brookfield Synchro-Electric viscometer, model LVT, with a No. 2 spindle at 12 rpm.

When air was used in the reservoir as the damping medium, a problem arose. The air in the reservoir was at atmospheric pressure. With the tunnel activated, the pressure over the test-plate area was naturally lower and the flexible skin would bulge upward. It was necessary, therefore, to attach a plastic tube from the reservoir outlet on the bottom of the test plate to a static pressure tap on the bottom of the tunnel floor in order to obtain a zero-pressure differential across the flexible skin. When this was done, the skin remained horizontal and did not bulge upward during the tests.

When fluids such as water and Polyox solutions were used as the damping mediums, the level in the test reservoir was achieved by routing the plastic tube from the reservoir to a sealed reservoir on the outside of the tunnel. The sealed reservoir's position was varied in the vertical direction to raise or lower the test reservoir level to the proper level. Another tube was tapped off the top of the sealed reservoir to the tunnel wall to provide reference pressure.

Boundary-layer data for a hard plate were obtained by inserting a hard wooden plate into the reservoir and then covering the entire test plate with a 0.0035-in. polyvinyl chloride sheet. Tension weights were hung on the sheet to eliminate any wrinkles on the surface of the plate.

Results and Discussions

Figures 3 and 4 are plots of Reynolds stresses measured with an X-array probe in the boundary layer. The physical size of the X-array probe limits the distance one can get near the wall. The data have been extrapolated to the wall in each figure in an attempt to get an indication of the wall shear stress. This extrapolation is based upon well-known experimental evidence that the total shear stress in a turbulent boundary layer is approximately equal to the Reynolds stress except in the thin viscous sublayer, where the Reynolds stress goes to zero, but the laminar stress increases to the final wall-skin-friction stress. The accuracy of the extrapolations is limited by the scatter of data and lack of data near the wall but it appears obvious that the flexible plate has lower Reynolds stresses than the hard plate. In addition, the skin-friction coefficient obtained from the Reynolds stresses in Fig. 4, near the middle of the plate (X_0 = 13 in.), appears to correlate closely with Looney's[17,20] measured value of the average coefficient.

Figures 2 and 5 are plots of the velocity profiles on both the hard and flexible plates. The hard plate data appear to fit the $\frac{1}{7}$ power law in Fig. 2 and the standard "Law of the

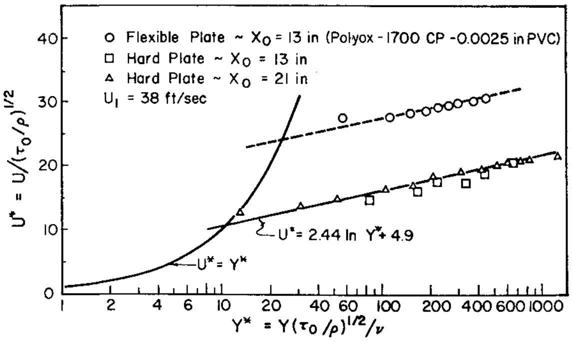

Fig. 5 Universal velocity profiles.

Wall" equation in Fig. 5. The extrapolated wall shear stress of Figs. 3 and 4 were used for the Y_0 in Fig. 5. The flexible wall data in Fig. 5 are parallel to the hard plate data which indicates that the Prandtl mixing length is the same for both hard and flexible plates. In addition, it appears that there is an increase in the laminar sublayer thickness of the flexible plate, if one assumed a priori that the laminar sublayer for the flexible plate follows the same equation as the hard plate ($U^* = Y^*$).

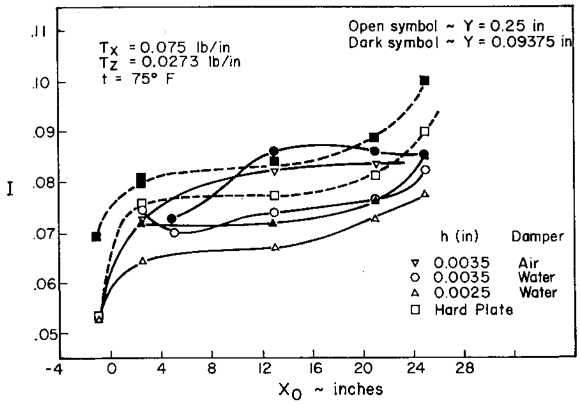

Fig. 6 Turbulence intensity variation with skin thickness and damping fluid.

One might speculate that the decrease in skin friction is somehow related to the increase in thickness of the laminar sublayer. This is exactly what was found by Wells[22] for pipe flow tests with dilute aqueous solutions of polymer additives. Wells found that along with a reduction in turbulent skin friction he had an increase in laminar sublayer thickness as his law of the wall plot for very dilute solutions appeared similar to the flexible wall data of Fig. 5.

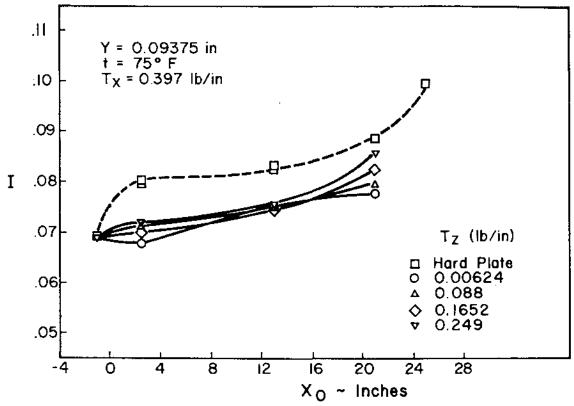

Fig. 8 Turbulence intensities for 0.0025-in. PVC skin with water as damping fluid.

It is not known if the turbulent boundary layer was in equilibrium. Just in front of the leading-edge gap ($X_0 = -1$ in.) a measured velocity profile indicated that the boundary layer was turbulent, since it closely approximated a $\frac{1}{7}$ velocity profile. The Reynolds number at the leading edge of the test plate was approximately 0.44×10^6. It is, therefore, probable that transition to a turbulent boundary layer occurred upstream of the leading edge, since normal flat plate transition Reynolds numbers range from 10^5 to 10^6. In addition, a 20° change in flow direction at the inlet nozzle-test section junction (at $X_0 = -15.5$ in.) probably helped to initiate transition.

Turbulence intensity measurements made along the length of the plate are shown in Figs. 6–11. From Fig. 6 it is seen that when air was used as the damping fluid in the reservoir, it did not damp out the turbulence as well as water did. As can be expected, it is seen that the turbulent intensities in the inner portion of the boundary layer (the dark symbols) are higher than in the outer portion. Figure 6 also indicates that the 0.0025-in.-thick polyvinyl skin damps out the turbulence better than the 0.0035-in.-thick skin. It is interesting to note that Looney[17,20] found the 0.0025-in.-thick skin has lower skin friction than the 0.0035-in.-thick skin, everything else being the same.

Another interesting thing to notice in Figs. 6–13 is that the turbulence intensity 1 in. in front of the leading edge of the test plate ($X_0 = -1$ in.) is considerably lower than the turbulence intensities over the test plate. The boundary-layer thickness increased from 0.51 in. at $X_0 = -1$ in. to 0.69 in. (for both hard plate and 1700-CP coating) at $X_0 = 2.5$ in. The boundary-layer thickness at $X_0 = 21$ in. was approximately 1.0 in. (see Fig. 2) for the hard plate. This indicates that the increase in boundary-layer thickness from $X_0 = -1$ in. to $X_0 = 2.5$ in. was abnormally high and was due

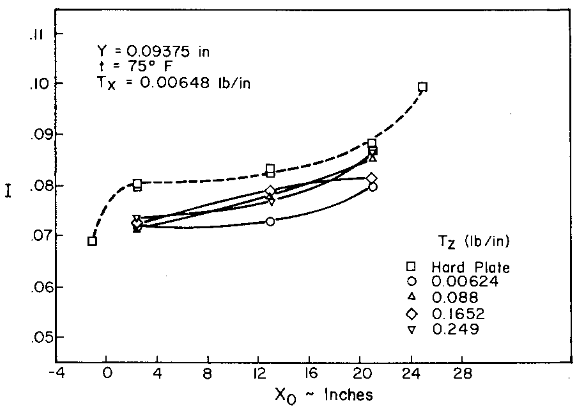

Fig. 7 Turbulence intensities for 0.0025-in. PVC skin with water as damping fluid.

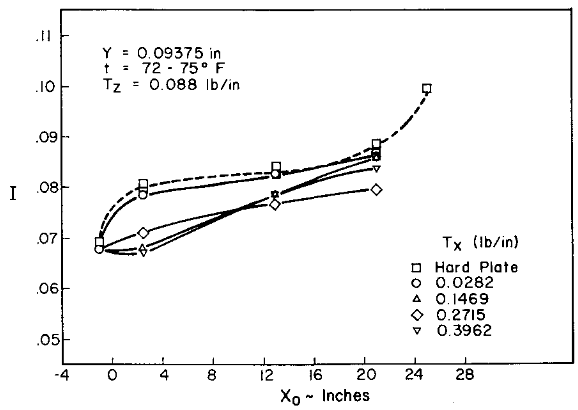

Fig. 9 Turbulence intensities for 0.0025-in. PVC skin with polyox 90 CP as damping fluid.

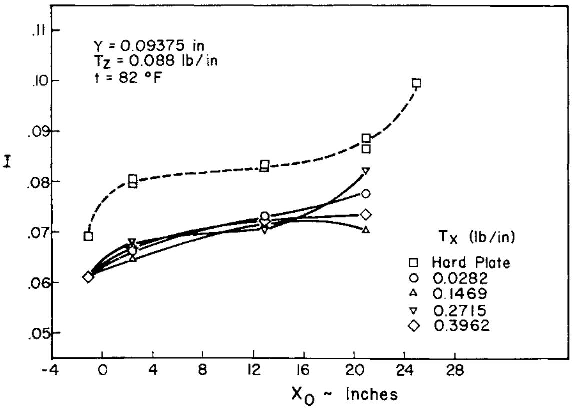

Fig. 10 Turbulence intensities for 0.0025-in. PVC skin with polyox 8000 CP as damping fluid.

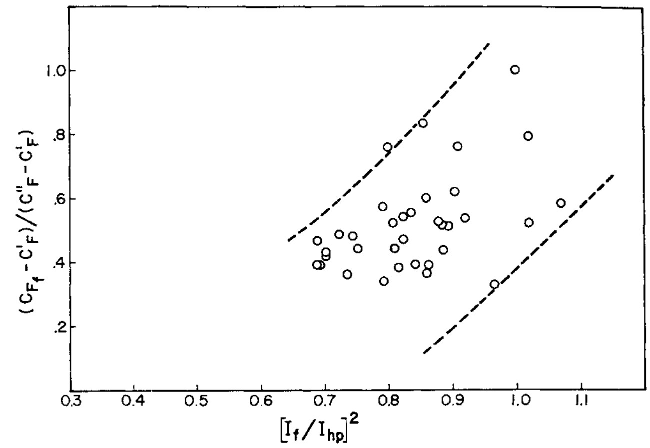

Fig. 12 Variation of average skin friction with average turbulence intensity.

primarily to the leading-edge gap. Now this abnormal increase in boundary-layer thickness would tend to decrease Y/δ for a fixed value of Y and cause an increase in turbulence intensity at a fixed value of $Y = 0.094$ since the peak in turbulence intensity is probably below $Y = 0.094$. Figures 7–11 of Hinze[18] indicate that turbulence intensity has a peak value at $Y/\delta = 0.0075$.

Figure 2 indicates that the boundary-layer thickness on a compliant coating is slightly less than on a hard plate at $X_0 = 21$ in. Thus, one might assume that the reason the turbulence intensities are lower for compliant coatings than hard plate at $X_0 = 21$ in. is because Y/δ is larger for a fixed Y, and hence, the compliant plate I data are measured further up from the I peak than hard-plate data. However, when one looks at the I data at $X_0 = 2.5$, where hard plate and compliant coatings had the same boundary thickness, we see that again the turbulence intensities for the compliant coating are smaller. Hence, the smaller values of turbulence intensities measured at fixed values of Y for compliant coatings are probably not due to the changes in boundary-layer thickness but due to some interaction mechanism between the fluid and the compliant surface that has not as yet been fully explained. It seems probable then that the main effect of the gap is to cause, for both the hard plate and compliant coating, a similar increase in the entire level of turbulence intensity throughout the boundary layer.

Figures 7 and 8 show the variation of turbulence intensities of a 0.0025-in. skin over water, with different skin tensions. It appears that there are only slight differences in turbulence intensities when the two different longitudinal tensions (T_x) are compared. The effect of the lateral tension is hard to state precisely, but it appears that, in general, the lower the value of T_z, the lower the turbulence intensity. Looney's[17,20] data show that for the same flexible surface the skin friction tended to decrease as the value of T_z decreased.

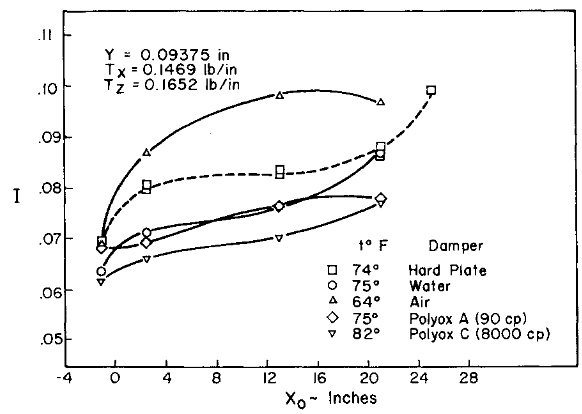

Fig. 11 Turbulence intensities for 0.0025-in. PVC skin with various damping fluids.

The data for the more viscous damping liquids are presented in Figs. 9 and 10. No discernible trend of longitudinal tension (T_x) vs turbulence intensities appears. However, in both figures the flexible skin plate appears to have lower turbulence intensity than the hard plate and the 8000-CP coating appears to be less affected by longitudinal tension (T_x).

Several of the different damping fluid coatings are compared in Fig. 11. One surprising result is the air data taken at 64°F. We have no explanation for the high air values. However, there is the possibility that the skin might have been stiffer at this lower temperature.

There is considerably more turbulence intensity data in the thesis of Walters[21]; however most of the trends found are illustrated by the figures in this paper. Large tension values were attempted on the polyvinyl chloride skin, but it was found that for the 0.0025-in.-thick skin permanent wrinkles appeared as the tension went up beyond 0.6 lb/in. In addition, low-tension values had to be avoided when using air as the damping fluid because the skin bowed up in the vertical direction quite readily when the tunnel was turned on. Only one size of test panel was used in these tests, so the effect of plate size and length/wide ratio will have to be determined from future tests.

Figure 12 was an attempt to verify experimentally the skin-friction-turbulence intensity relation predicted by Eq. (1). The skin-friction coefficients were taken from Looney's data[11,22] and the turbulence intensity data were taken from Walter's thesis.[21] It should be noted that although Eq. (1) shows a relationship between local skin friction and local turbulence intensity, average skin friction is plotted vs average turbulence intensity in Fig. 12. Average values were used in Fig. 12 because no experimental local skin-friction coefficient data were available. It is obvious from Fig. 12 that there is considerable scatter of data and the correlation between skin-friction reduction and turbulence intensity reduction is only fair. However, in all but two or three cases, a reduction in skin friction was accompanied by a measured reduction in turbulence intensity. The real basis for skin-friction reduction is still far from being completely understood, but these experiments have shown that it is very probably a function of the amount of turbulence damping performed by the compliant coating.

References

[1] Kramer, M. O., "Boundary Layer Stabilization by Distributed Damping," *The Journal of Aeronautical Sciences*, Vol. 24, 1957, p. 459.

[2] Kramer, M. O., "Boundary Layer Stabilization by Distributed Damping," *Journal of the American Society of Naval Engineers*, Vol. 72, 1960, pp. 25–33.

[3] Kramer, M. O., "The Dolphin's Secret," *Journal of the American Society of Naval Engineers*, Vol. 73, 1961, pp. 103–107.

[4] Benjamin, T. B., "Effects of a Flexible Boundary on Hydrodynamic Stability," *Journal of Fluid Mechanics*, Vol. 9, 1960, p. 513.

[5] Landahl, M. T., "On the Stability of a Laminar Incompressible Boundary Layer over a Flexible Surface," *Journal of Fluid Mechanics*, Vol. 13, 1962, pp. 609–632.

[6] Kaplan, R. E., "The Stability of Laminar Incompressible Boundary Layers in the Presence of Compliant Boundaries," TR 116-1, June 1964, Aeroelastic and Structures Research Lab., Massachusetts Institute of Technology, Cambridge, Mass.

[7] Karplus, H. B., "Turbulent Flow Transition near Solid and Flexible Boundaries," Rept. IITRI 1205-4, March 1963, Illinois Institute of Technology Research Institute, Chicago, Ill.

[8] Laufer, J. and Maestrello, L. "The Turbulent Boundary Layer over a Flexible Surface," Document D6-9708, 1963, The Boeing Co. Transport Division.

[9] Smith, L. L. "An Experiment on Turbulent Flow in a Pipe with a Flexible Wall," Master of Science thesis, 1963, Dept. of Chemical Engineering, University of Washington.

[10] Gregory, N. and Love, E. M. "Progress Report on an Experiment of the Effect of Surface Flexibility on the Stability of Laminar Flow," CP 602, 1961, Aeronautical Research Council, Ministry of Aviation, London, England.

[11] Benjamin, T. B., "Fluid Flow with Flexible Boundaries," Eleventh International Congress of Applied Mechanics, Munich, 1964.

[12] Stephens, A. V., private communication, May 4, 1966, Dept. of Aeronautical Engineering, The Queens University of Belfast.

[13] Ritter, H. and Messum, L. T., "Water Tunnel Measurements of Turbulent Skin Friction on Six Different Compliant Surfaces of 1 Ft Length," ARL/N4/GHY/9/7, ARL/G/N9, June 1964, Admiralty Research Lab.

[14] Von Winkel, W. A. and Barger, J. E., "Evaluation of a Boundary Layer Stabilization Coating," *Journal of the Acoustical Society of America*, Vol. 33, 1961, p. 836.

[15] Pelt, R., "A Preliminary Investigation of Surface Damping Effects on Fluid Flow through Flexible Tubes," Ph.D. dissertation, 1964, Dept. of Chemical Engineering, University of Pittsburgh, Pittsburgh, Pa.

[16] Fisher, D. H. and Blick, E. F., "Turbulent Damping by Flabby Skins," *Journal of Aircraft*, Vol. 3, No. 2, March–April 1966, pp. 163–164.

[17] Looney, R. W. and Blick, E. F., "Skin Friction Coefficients of Compliant Surfaces in Turbulent Flow," *Journal of Spacecraft and Rockets*, Vol. 3, No. 3, Oct. 1966, pp. 1562–1564.

[18] Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959, pp. 491–92.

[19] Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," Rept. 1247, 1955, NACA, Washington, D. C.

[20] Looney, R. W., "Skin Friction Coefficients of Compliant Surfaces in Turbulent Flow," Master of Engineering thesis, 1966, School of Aerospace and Mechanical Engineering, University of Oklahoma, Norman, Okla.

[21] Walters, R., "Experiments on Aerodynamic Turbulent Damping of a Flat Membrane with Fluid Substrate," Master of Engineering thesis, 1966, School of Aerospace and Mechanical Engineering, University of Oklahoma, Norman, Okla.

[22] Wells, C. S., Jr., "Anomalous Turbulent Flow of Non-Newtonian Fluids," *AIAA Journal*, Vol. 3, No. 3, Oct. 1965, pp. 1800–1805.